Fermi National Accelerator Laboratory

Patterns from Preheating

Andrew Sornborger and Matthew Parry

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

December 1998

Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Distribution

Approved for public release; further dissemination unlimited.

Copyright Notification

This manuscript has been authored by Universities Research Association, Inc. under contract No. DE-AC02-76CHO3000 with the U.S. Department of Energy. The United States Government and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government Purposes.

Patterns from Preheating

A. Sornborger

NASA/Fermilab Astrophysics Group, Fermi National Accelerator Laboratory, Box 500, Batavia, IL 60510-0500, USA

M. Parry

Department of Physics, Brown University, Providence, RI 02912

(November 19, 1998)

The formation of regular patterns is a well-known phenomenon in condensed matter physics. Systems that exhibit pattern formation are typically driven and dissipative with pattern formation occurring in the weakly non-linear regime and sometimes even in more strongly non-linear regions of parameter space. In the early universe, parametric resonance can drive explosive particle production called preheating. The fields that are populated then decay quantum mechanically if their particles are unstable. Thus, during preheating, a driven-dissipative system exists. In this paper, we show that a self-coupled inflaton oscillating in its potential at the end of inflation can exhibit pattern formation.

Much recent work has been done on the topic of preheating in inflationary cosmology. Preheating is a stage of explosive particle production which results from the resonant driving of particle modes by an inflaton oscillating in its potential at the end of inflation [1–3].

In regions of parameter space where parametric resonance is effective, much of the energy of the inflaton is transferred to bands of resonant wave modes. This energy transfer is non-thermal and can lead to interesting non-equilibrium behavior. Two examples of the non-equilibrium effects that can be produced are non-thermal phase transitions [4–7] and baryogenesis [8,9]. The non-thermal phase transitions induced during preheating can also possibly lead to topological defect formation [10–12], even at energies above the eventual final thermal temperature. Furthermore, non-linear evolution of the field when quantum decay of the resonantly produced particles is small leads to a turbulent power-law spectrum of density fluctuations [13,15].

In this letter, we present a new phenomenon that can arise from preheating: pattern formation. It has long been known that many condensed matter systems exhibit pattern formation*. Examples of pattern forming systems which have been studied are ripples on sand dunes, cloud streets and a variety of other convective systems, chemical reaction-diffusion systems, stellar atmospheres and vibrated granular materials. All of these physical systems have two features in common. They are all driven in some manner, i.e. energy is input to the system, and they are all dissipative, usually being governed by diffusive equations of motion. Typically, patterns are formed in these systems in the weakly non-linear regime before the energy introduced into the system overwhelms the dissipative mechanism. Sometimes, patterns persist beyond the weakly non-linear regime as well.

At the end of inflation, the inflaton ϕ is homogeneous with small perturbations $\delta\phi$ imprinted on it due to quantum fluctuations. In chaotic inflationary models, the inflaton then oscillates about the minimum of its potential, giving an effectively time dependent mass to fields with which it is coupled. The time dependent mass drives exponential growth in the population of bands of particle wave modes. Many of the fields into which the inflaton can decay resonantly are also unstable to quantum decay. For these reasons, at the end of inflation, we are considering fields which are driven, due to resonant particle creation, and also dissipative, due to quantum decay. Therefore, it makes sense to see if there is a region of parameter space where the system is in the weakly non-linear regime and exhibits pattern formation.

In our investigation we consider $\lambda\phi^4$ theory with the addition of a phenomenological decay term to mimic the inflaton's quantum decay. This model without the decay term has been studied extensively in the literature [2,3,13–16,19], and a similar model including the decay term has also been studied [20].

Our field equation is

$$\ddot{\phi} + \gamma\dot{\phi} - \nabla^2\phi + \lambda\phi^3 = 0 \qquad (1)$$

where γ is a decay constant and λ is the self-coupling of the field. Here, we neglect the expansion of the universe, although we will comment on the effect of expansion later. For our calculations, we rescale: $t \to t/\sqrt{\lambda}\phi_0$, $x \to x/\sqrt{\lambda}\phi_0$ and $\phi \to \phi\,\phi_0$, where ϕ_0 is the value of

---

*For an extensive review of pattern formation in condensed matter systems, see [17,18]

the inflaton at the end of inflation. This gives us a new equation

$$\ddot{\phi} + \Gamma\dot{\phi} - \nabla^2\phi + \phi^3 = 0 \qquad (2)$$

where $\Gamma = \gamma/\sqrt{\lambda}\phi_0$.

It should be noted that pattern formation in the inflaton system is conceptually distinct from condensed matter systems for at least two reasons. First, the equations we study are wave equations with damping, not diffusive equations. Secondly, we expect wave patterns to be formed while the homogeneous mode decays, therefore pattern formation will be a temporary phenomenon, at least in the model above in which gravity is neglected. This should be considered in contrast to the typical condensed matter system, in which energy is introduced via boundary conditions (in a convective system) or by a vibrating bed (in a granular material system), and the energy input is essentially constant.

For $\Gamma = 0$, the resonant modes lie in the interval

$$\frac{3}{2} < k^2 < \sqrt{3}. \qquad (3)$$

For $\Gamma \neq 0$, we can introduce $\varphi = \phi\, e^{\frac{\Gamma}{2}t}$ giving

$$\ddot{\varphi} - (\nabla^2 + \frac{\Gamma^2}{4})\varphi + e^{-\Gamma t}\varphi^3 = 0. \qquad (4)$$

Therefore, for small Γ, we expect the resonance bands to be slightly shifted, because the ∇^2 term is shifted by $\Gamma^2/4$, and we expect the resonance to diminish slowly over time, due to the exponential damping of the potential with time. It should be noted however that the resonance structure of the equation can be quite sensitive to changes in the potential [19].

As the effect that we are trying to isolate is non-linear, we resort to numerical simulation of the field equation. We use a leapfrog code which is second-order accurate in time and we use fourth-order spatial differences. We simulate the field in two dimensions in a box with periodic boundary conditions which has 256 grid points per dimension.

For initial conditions, we give small amplitudes ($\sim 10^{-3}$) to all resonant modes in the box. It is unnecessary to populate non-resonant modes because they will decay while the resonant modes grow. In the initial condition setup, it is crucial that there be many resonant modes in the box.

Early on, we found spurious pattern formation when we set the box size such that the resonant mode wave number in the box was 3. For such small wave numbers, only two resonant modes exist in the box, one along the x-axis, and one along the y-axis, giving rise to a misleading square wave pattern. Setting the box size such that the resonant wave number in the box is 16 is enough to give many different resonant modes in the box, but still have

good resolution of the wave, so this is the box size we used.

We set out to identify the weakly non-linear regime. In [13] it is shown that the self-coupled inflaton system's non-linear time evolution proceeds as follows: First, the resonant band amplitude grows. Next, when the amplitude in the resonant band is high enough for non-linear effects to become important, period doubling occurs and subsidiary peaks develop in the power spectrum. Further peaks then develop and the spectrum broadens and approaches a power-law spectrum. We tuned Γ such that the amplitude of the resonant band grew, but little period doubling occurred. In this regime, only resonant mode wavelengths would exist in the box and they would interact with each other non-linearly.

In figure 1 we plot a superposition of the power spectrum at various times during our simulation. It is possible to see that for the value $\Gamma = 0.0035$, the system stays in the weakly non-linear regime for the entire simulation.
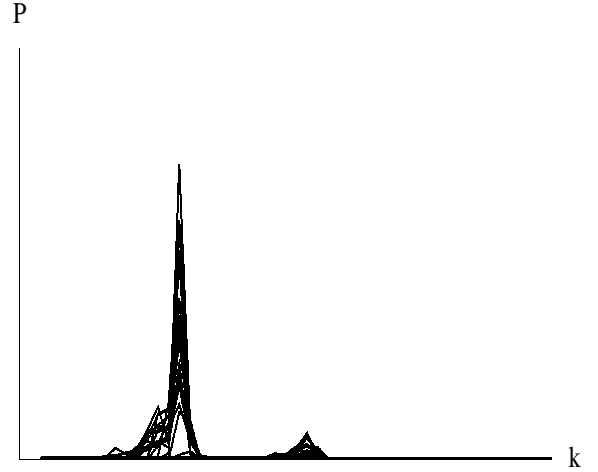

FIG. 1. A superposition of power spectra taken from a simulation with $\Gamma = 0.0035$. We plot the amplitude of the power spectra vs. wave number. Note that the period doubling modes are only weakly populated, indicating that the simulation is in the weakly non-linear regime.

When wave patterns form, the specific pattern which arises is due to the non-linear interaction of the wave modes. The amplitude of wave modes separated by different angles grows at different rates. Modes separated by angles with the fastest growing amplitudes dominate the solution and form the wave pattern. In plots 2, 3, 4 and 5, we show the evolution of the Fourier transform of the inflaton. In the final plot, it is clear that the dominant modes have picked out a preferred configuration in Fourier space. In figure 6, we also plot $\phi(x, y)$ in configuration space at the time when the pattern has formed. It should be commented that the preferred angles in Fourier space of the pattern would pick out a dodecahedron, but this is not a close-packable structure, therefore the pattern looks more complicated in configuration space, than

2

in Fourier space. After this time, the field amplitude begins to decay; the wave pattern remains imprinted, but decreases in amplitude.
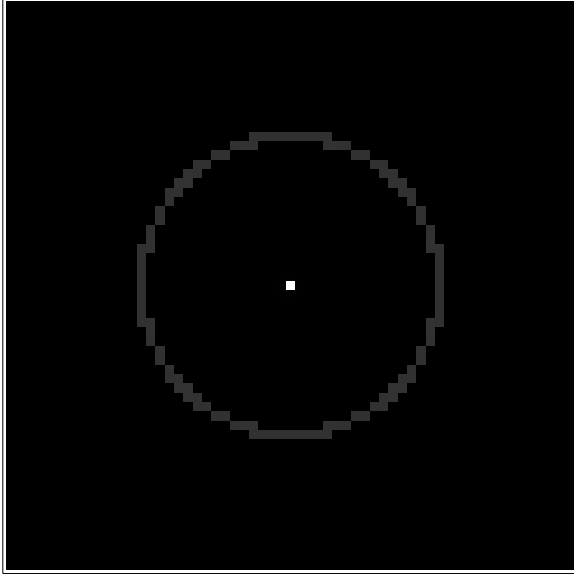

FIG. 2. Initial conditions for the simulation ($\Gamma = 0.0035$). $\tilde{\phi}(k_x, k_y)$, the Fourier transform of the inflaton is plotted. The central peak is the zero mode, and the surrounding ring is the resonant mode populated with small amplitudes. $t = 0$. Only the region of interest is plotted. All modes outside of the plotted region have zero amplitude.
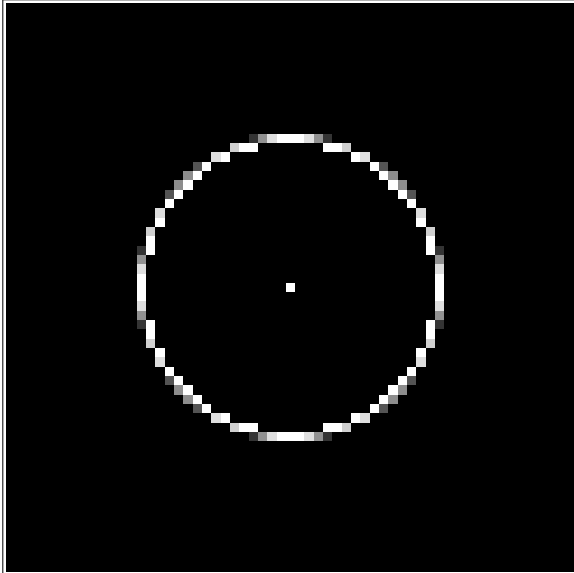

FIG. 3. After resonance begins to boost the amplitude of the resonant mode. Notice the brightening (increasing amplitude) of the resonant modes. $t = 60$.
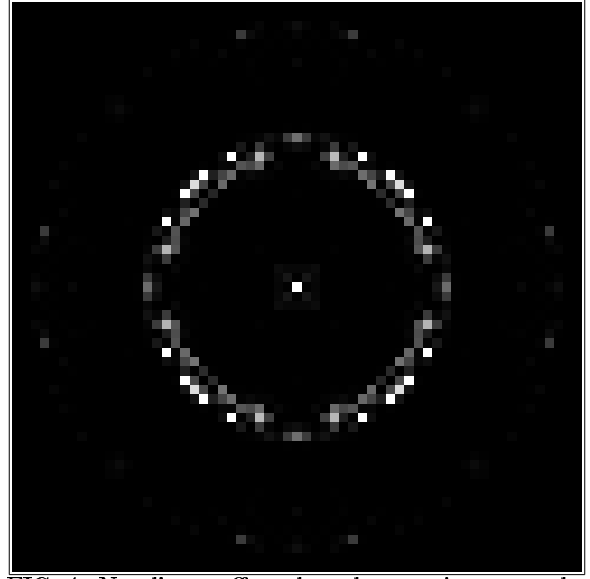

FIG. 4. Non-linear effects have become important by this time causing fragmenting of the resonant mode. Modes with dominant growth are taking over. $t = 210$.
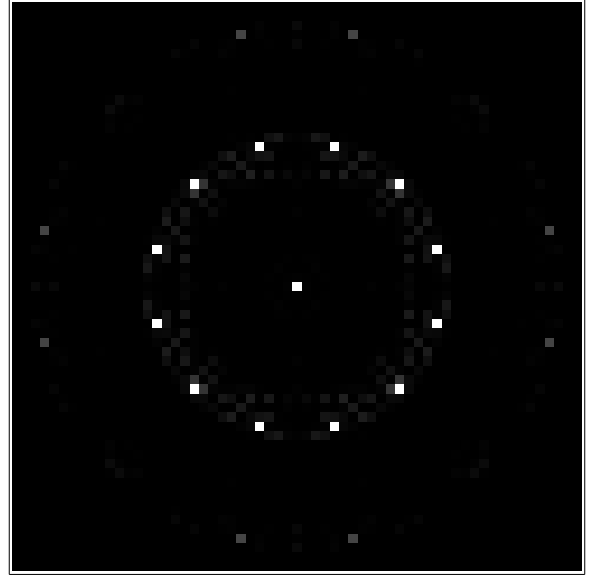

FIG. 5. The final wave pattern. $t = 300$.

The important result here is not that we have found the preferred wave pattern of the system. With only a discrete set of wavemodes in the box, we cannot be sure that some other pattern would not dominate if we were able to investigate a continuum of modes. However, it is clear that some pattern will be picked out by the system in this parameter region. Therefore, we can claim to have identified pattern formation as a phenomenon exhibited by the damped, self-coupled inflaton.

It would also seem that pattern formation should exist in realizations of parametric resonance and preheating other than the system we have analyzed, simply because of their driven-dissipative nature.

3

In an expanding universe, after a similar variable and field transformation as before, the field equations (in conformal time) become

$$\ddot{\varphi} + a\Gamma\dot{\varphi} - \nabla^2\varphi - (\dot{a}\Gamma + \frac{\ddot{a}}{a})\varphi + \varphi^3 = 0, \qquad (5)$$

where here $\varphi = a\phi$. Note that there is no Hubble damping term because the theory without damping due to quantum decay is conformally invariant. (We are studying other models though, to see whether Hubble damping can provide the necessary damping for pattern formation.) For the expanding case, we expect pattern formation for smaller values of Γ as the effective dissipation grows with the scale factor a. Oscillation of the homogeneous mode should be dominated by the φ^3 term at early times before φ has had time to be significantly damped. We are currently investigating this case.
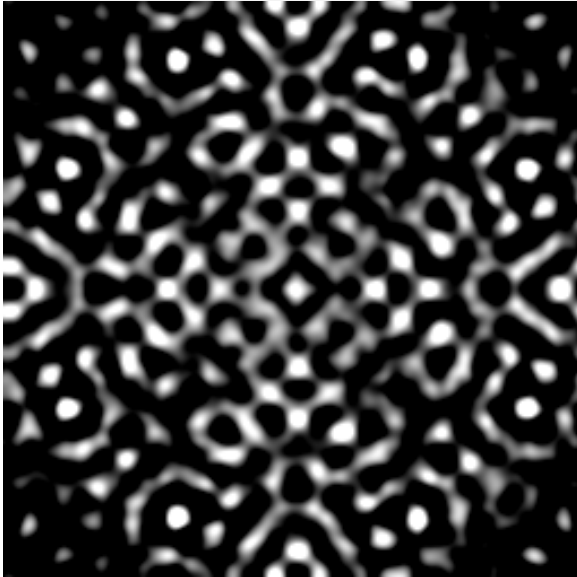

FIG. 6. The final wave pattern in configuration space. $t = 300$. The corners are darkened by small amplitude long wavelength modes which modulate the wave pattern.

For wave patterns to have an impact cosmologically, they must persist after they are created. In our toy model, the resonance band corresponds to wavelengths roughly the size of the Hubble radius at the start of preheating, about $10^{-25}\,cm$. The size of these fluctuations today would then only be about $10\,m$. However we have seen that the patterns are modulated by long wavelength modes, and it is this non-linear transfer of power to small k-modes that may be important. Thus the size of the Hubble radius at the *end* of preheating may actually set the scale for wave patterns. In any case, as the wave patterns are regions of energy density and are present essentially from the beginning of the universe, they may be sites of significant gravitational accretion. As dark matter seeds it may be possible for wave patterns to change the thermal history of baryonic matter. It is this di-

rection we intend to take in determining the role wave patterns may play in the history of the universe.

[1] J. H. Traschen & R. H. Brandenberger, *Phys. Rev. D* **42**, (1990) 2491.
[2] L. A. Kofman, A. D. Linde & A. A. Starobinsky, *Phys. Rev. Lett.* **73**, (1994) 3195.
[3] Y. Shtanov, J. Traschen & R. Brandenberger, *Phys. Rev. D* **51**, (1995) 5438.
[4] L. Kofman, A. Linde & A. Starobinsky, *Phys. Rev. Lett.* **76**, (1996) 1011.
[5] I. Tkachev, *Phys. Lett. B* **376**, (1996) 35.
[6] A. Riotto & I. Tkachev, *Phys. Lett. B* **385**, (1996) 57.
[7] S. Khlebnikov, L. Kofman, A. Linde & I. Tkachev, *Phys. Rev. Lett.* **81**, (1998) 2012.
[8] G. W. Anderson, A. Linde & A. Riotto, *Phys. Rev. Lett.* **77**, (1996) 3716.
[9] E. W. Kolb, A. Linde & A. Riotto, *Phys. Rev. Lett.* **77**, (1996) 4290.
[10] S. Kasuya & M. Kawasaki, *Phys. Rev. D* **56**, (1997) 7597.
[11] I. Tkachev, S. Khlebnikov, L. Kofman & A. Linde, hep-ph/9805209.
[12] M. Parry & A. Sornborger, hep-ph/9805211.
[13] S. Khlebnikov & I. Tkachev, *Phys. Rev. Lett.* **77**, (1996) 219.
[14] D. Boyanovsky, H. J. de Vega, R. Holman & J. F. J. Salgado, *Phys. Rev. D* **54**, (1996), 7570.
[15] T. Prokopec & T. G. Roos, *Phys. Rev. D* **55**, (1997) 3768.
[16] D. I. Kaiser, *Phys. Rev. D* **56**, (1997) 706.
[17] M. C. Cross & P. C. Hohenberg, *Rev. Mod. Phys.* **65**, (1993) 851.
[18] A. Newell, T. Passot & J. Lega, *Ann. Rev. Fluid Mech.* **25**, (1993) 399.
[19] P. B. Greene, L. Kofman, A. Linde & A. Starobinsky, *Phys. Rev. D* **56**, (1997) 6175.
[20] E. W. Kolb, A. Riotto & I. Tkachev, *Phys. Lett. B* **423**, (1998) 348.